ROTATION OF A CYLINDER IN A VISCOUS LIQUID

A. I. Khasanov

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1. Introduction. Let a viscous incompressible liquid fill the space outside an infinite cylinder of radius a. We will consider the rotationally symmetric motion of the liquid produced by rotation of the cylinder about its own axis at an angular velocity ω. We introduce a cylindrical coordinate system with z axis coinciding with the cylinder axis. It is known that in such coordinates the Navier–Stokes equation admits a solution of the form $u = f(r)$, $v = g(r)$, $w = -zh(r)$, $p = K\rho z^2/2 + \varphi(r)$, where u, v, w are the velocity components, p is pressure, ρ is liquid density, K is some constant, and the functions f, g, h, φ are defined by the equations

$$\nu(f''' + 2f''/r - f'/r^2 + f/r^3) - ff'' + f'^2 + ff'/r + 2f^2/r^2 = K; \tag{1.1}$$

$$\nu(g'' + g'/r - g/r^2) - f(g' + g/r) = 0; \tag{1.2}$$

$$h - f' - f/r = 0; \tag{1.3}$$

$$\varphi' - \nu(f'' + f'/r - f/r^2) + ff' - g^2/r = 0. \tag{1.4}$$

The condition of adhesion to the cylinder requires that

$$f(a) = 0,\ g(a) = \omega a,\ h(a) = 0. \tag{1.5}$$

Equations (1.3), (1.5) determine boundary conditions for Eq. (1.1) in the form

$$f(a) = f'(a) = 0. \tag{1.6}$$

If the solution of Eqs. (1.1), (1.6) is known, then the functions φ, g, h can be found from Eqs. (1.2)-(1.4) with the aid of quadratures. We note that Eq. (1.2) has a solution $g = \omega a^2/r$, which at K = 0 and f = 0 corresponds to velocity and pressure fields consistent with the adhesion conditions

$$u = 0,\ v = \omega a^2/r,\ w = 0,\ p = -\rho\omega a^2/2r^2 + \text{const}. \tag{1.7}$$

The present study will prove the existence of solutions of the problem of Eqs. (1.1)-(1.6) differing from Eq. (1.7), and will clarify the behavior of such solutions as $\nu \to 0$.

2. Evaluation of Conditions at Infinity. Since we deal with an infinite region, to define the solutions of Eqs. (1.1)-(1.6), it is necessary to formulate conditions at infinity.

In Eq. (1.1) we take K = 0 and require that

$$fr \to 0 \text{ as } r \to \infty. \tag{2.1}$$

Then the boundary-problem equations (1.1), (1.6), (2.1) has a unique solution $f \equiv 0$. We will prove this fact. If we make the replacement $u = rf(r)$, then from Eqs. (1.1), (1.6), (2.1) we obtain the following boundary problem:

$$\nu(u''' + u'/r^2 - u''/r) - uu''/r + u'^2/r + uu'/r^2 = 0,$$
$$u(a) = u'(a) = 0,\ u \to 0,\ r \to \infty.$$

We note that $(\nu(u'' - u'/r) - uu'/r)' = -2u'^2/r \leqslant 0$, whence we obtain the inequality

$$\nu(u'' - u'/r) - uu'/r \leqslant \nu u''(a). \tag{2.2}$$

For $u''(a) \leqslant 0$, we obtain from Eq. (2.2) with consideration of $u'(a) = 0$ that $u'(r) \leqslant 0$, whence follows the required $u \equiv 0$. For $u''(a) > 0$ either $u'(r) \geqslant 0$ for any $r \geqslant a$, or else there exists an $r_0 > a$ such that $u'(r_0) = 0$, $u''(r_0) < 0$ and $u'(r) \geqslant 0$ for $a \leqslant r \leqslant r_0$. In the second case, for $r \geqslant r_0$ we have

$$\nu(u'' - u'/r) - uu'/r \leqslant 0. \tag{2.3}$$

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Integration of Eq. (2.3) with consideration of $u'(r_0) = 0$ gives $u'(r) \leqslant 0$ for $r \geqslant r_0$. On the other hand, $u \to 0$ as $r \to \infty$, so that $u' \to 0$ as $r \to \infty$ and $u \geqslant 0$ for $r \leqslant r_0$. From inequality (2.3) it follows that $\nu u'' \leqslant uu' + \nu u'/r \leq 0$ on the interval (r_0, ∞), which implies that $u'(r) \equiv 0$ on the same interval in view of the boundary conditions for $u'(r)$. This contradicts our choice of the point r_0, whence we conclude that the sign of $u'(r)$ is defined over the entire interval (a, ∞) so that $u \equiv 0$, and the case $u''(a) > 0$ is not realizable.

In Eq. (1.1) let $K > 0$. Then the problem of Eqs. (1.1), (1.6), (2.1) has no solution. In fact, in this case inequality (2.2) remains in force. Therefore, this problem can have no solution other than the trivial $f \equiv 0$. But $f \equiv 0$ is not a solution at $K \neq 0$.

For the case $K < 0$ we will show the existence of solutions of Eqs. (1.1), (1.6), satisfying the condition

$$f' \to -c \quad \text{at} \quad r \to \infty, \tag{2.4}$$

where $c = \sqrt{K/4}$.

3. Existence Theorem. The proof of existence of a solution of boundary problem (1.1), (1.6), (2.4) rests on a lemma concerning solutions of ordinary differential equations [1].

We will give the definition necessary for formulation of the lemma.

Definition 1. Let y, G be n-dimensional vectors with the function $G(t, y)$ continuous in the open set Ω of an Euclidean space of dimensionality $n + 1$. Let Ω_0 be an open subset of Ω. We denote by $\partial\Omega_0$ and $\bar{\Omega}_0$, respectively, the boundary and closing of the set. Then $(t_0, y_0) \in \partial\Omega_0$ and $\bar{\Omega}_0$ is termed the exit (or entry) point for the set Ω_0 in relation to the system

$$dy/dt = G(t, y), \tag{3.1}$$

if for each solution $y(t)$ of this system satisfying the condition $y(t_0) = y_0$, there exists a $\delta > 0$ such that $(t, y(t)) \in \Omega_0$ for $t_0 - \delta < t < t_0 (t_0 < t < t_0 + \delta)$. If, in addition $(t, y(t)) \notin \bar{\Omega}_0$ at $t_0 < t < t_0 + \delta (t_0 - \delta < t < t_0)$, then the point (t_0, y_0) is termed a strict exit (entry) point.

LEMMA 1. Under the conditions of Definition 1, let the function $G(t, y)$ be such that the Cauchy problem for system (3.1) has a unique solution, while the set Ω_0 is such that all exit points are strict exit points and the set Ω_e of exit points is not bound. We denote by Ω_i the set of entry points for Ω_0, and S is an affine subset within $\Omega_0 \cup \Omega_e \cup \Omega_i$, such that $S \cap (\Omega_0 \cup \Omega_i)$ contains two points (t_1, y_1) and (t_2, y_2) for which the solutions of system (3.1) passing through (t_j, y_j) at $j = 1, 2$ exit Ω_0 with growth in t at points belonging to different affine components of the set Ω_0. Then there can be found at least one point $(t_0, y_0) \in S \cap (\Omega_0 \cup \Omega_i)$, such that the solution $y_0(t)$ of system (3.1) passing through (t_0, y_0), remains in Ω_e at its right-hand maximum interval of existence.

We will now turn to proof of the solubility of the problem formulated. With the replacement of variables

$$t = r, \quad y_1 = -f, \quad y_2 = y_1' + r^{-1} y_1, \quad y_3 = y_2' \tag{3.2}$$

we reduce Eq. (1.1) to a system of first-order equations

$$dy_1/dt = y_2 - y_1/t, \quad dy_2/dt = y_3, \quad dy_3/dt = -y_3/t + y_2^2 - y_1 y_3 - 4c^2. \tag{3.3}$$

The boundary conditions have the form

$$y_1(a) = y_2(a) = 0, \ y_2 \to 2c \quad \text{as} \quad t \to \infty.$$

We then take

$$\begin{aligned}
\Omega_0 &= \{(t, y): t > a, \ y_1 \ \text{arbitrary}, \quad 0 < y_2 < 2c, \ y_3 > 0\},\\
\Omega^1 &= \{(t, y): t > a, \ y_1 \ \text{arbitrary}, \quad 0 < y_2 < 2c, \ y_3 = 0\},\\
\Omega^2 &= \{(t, y): t > a, \ y_1 \ \text{"} \quad y_2 = 2c, \ y_3 > 0\},\\
\Omega^3 &= \{(t, y): t > a, \ y_1 \ \text{"} \quad y_2 = 2c, \ y_3 = 0\},\\
\Omega_i &= \{(t, y): t > a, \ y_1 \ \text{"} \quad y_2 = 0, \ y_3 > 0\}.
\end{aligned}$$

The set Ω_0 is open, and the entry points into Ω_0 form a set Ω_i. To apply the lemma formulated above, we make use of the fact that system (3.3) has a solution $y_1 = cr + \alpha r^{-1}$, $y_2 = 2c$, $y_3 = 0$, where α is arbitrary. This solution corresponds to points from Ω^3, and therefore they are neither entry not exit points. The set of exit points from Ω_0 coincides with $\Omega^1 \cup \Omega^2$, since in Ω^1 $y_3' < 0$, and in Ω^2 $y_2' = y_3 > 0$.

The set $\Omega_e = \Omega^1 \cup \Omega^2$ proves to be nonaffine. We denote by S the affine subset $\Omega_0 \cup \Omega_i$, defined as $S = \{t = a, (0, 0, \gamma) = y\}$, where $\gamma > 0$.

Let $y_\gamma(t)$ be a solution of system (3.3) passing through the point $(t, y) = (a, 0, 0, \gamma)$. From the third equation of system (3.3) it follows that $y_3'(a) < -4c^2$, so that $y_3'(t) < -2c^2$ for $a \leqslant t \leqslant a + \delta$, $\delta > 0$. Choosing $\gamma < \min(2c^2\delta, 2c/\delta)$, we obtain $y_3(a + \delta) \leqslant \gamma - 2c^2\delta < 0$, while $y_2(t) < \gamma(t - a) < \gamma\delta < 2c$, since $y_2' = y_3 < \gamma$, and this means that at some $\leqslant t_1 \leqslant a + \delta)$ the curve $(t, y_\gamma(t))$ exits Ω_0 at a point from Ω^1.

The third equation of Eq. (3.3) can be written as

$$\nu(ty_3)' = (-ty_1y_2)' + 2ty_2^2 - 4c^2t. \tag{3.4}$$

Since $2ty_2^2 \geqslant 0$, by estimating a lower limit for the right side of Eq. (3.4) and integrating along the curve $(t, y_\gamma(t))$, we obtain $\nu ty_3 \geqslant \nu a\gamma + 2c^2a^2 - 2t^2c^2 - ty_1y_2$. Fixing $t = T$ and choosing γ sufficiently large, it can be shown that $y_3 \geqslant a\gamma/2 > 0$ on the interval $a \leqslant t \leqslant T$ and $y_2(T) \geqslant a\gamma T/2 > 2c$. Thus the curve $(t, y_\gamma(t))$ exits from Ω_0 at a point where $y_2 = 2c$.

From Lemma 1 we find $\gamma = \gamma_0 > 0$ such that the solution $y_{\gamma_0}(t)$ remains in Ω_0 at its right-hand maximum interval of existence. Since $0 \leqslant y_2 \leqslant 2c$, this interval coincides with the half-line $t \geqslant a$. We will show that $y_2 \to 2c$ as $t \to \infty$.

Since $y_2 > 0$ at $t > a$, it then follows that $y_1 > 0$ at $t > a$ (first equation of Eq. (3.3)), then $y_3 > 0$ and $y_2 < 2c$ give $y_3' < 0$. Therefore, there exists a limit $y_3(t)$ as $t \to \infty$, which is equal to zero because of the finiteness of y_2. This means that $y_2 \to 2c$ as $t \to \infty$ (in the contrary case $y_3' < -\text{const} < 0$). The existence of a solution to the problem has been proven.

4. Analysis of Solution Asymptotes. In the new variables τ, $x = (x_1, x_2, x_3)$, such that

$$t = a + \sqrt{\nu/c}\,\tau,\ y_1 = \sqrt{\nu c}\,x_1,\ y_2 = cx_2,\ y_3 = c\sqrt{c/\nu}\,x_3, \tag{4.1}$$

the problem of Eqs. (1.1), (1.6), (2.4) has the form

$$dx_1/d\tau = x_2 - \varepsilon x_1/(1 + \varepsilon\tau),\ dx_2/d\tau = x_3, \tag{4.2}$$

$dx_3/d\tau = -\varepsilon x_3/(1 + \varepsilon\tau) - x_1x_3 + x_2^2 - 4$, $x_1(0) = x_2(0) = 0$; $x_2 \to 2$, $\tau \to \infty$, where $\varepsilon = R^{-1/2}$; $R = a^2c/\nu$ is a parameter playing the role of the Reynolds number.

We will show that for $\varepsilon \leqslant 1$ it is true that

$$1 < C_1 \leqslant x_3(0) \leqslant C_2. \tag{4.3}$$

In view of the choice of the set Ω_0 made in proving the existence theorem and the transformation formulas (4.1) we have the inequalities

$$0 < x_2 < 2,\ x_1 \geqslant 0,\ x_3 > 0. \tag{4.4}$$

In view of Eq. (4.4), the third equation of Eq. (4.2) gives $x_3' < 0$, whence $x_3 \leq x_3(0) = \gamma_0$. Then the second equation of Eq. (4.2) permits us to obtain $x_2 < \gamma_0$ at $0 \leqslant \tau < 1$. Therefore, at the same τ $x_3' < \gamma_0^2 - 4$, and consequently, $x_3 < \gamma_0 + \gamma_0^2 - 4$. Since $x_3 > 0$, then $\gamma_0 + \gamma_0^2 - 4 > 0$. From this inequality it follows that $\gamma_0 \geq C_1 > 1$.

From the first equation of Eq. (4.2) with the use of inequalities (4.4) we can obtain a lower limit for $x_3(1)$: $x_3(1) \leqslant -8 - 2\varepsilon + \gamma_0/(1 + \varepsilon)$. Inasmuch as $x_3(\tau) > x_3(1)$ at $0 \leq \tau < 1$, we find that $x_2(\tau) > -8 - 2\varepsilon + \gamma_0/(1 + \varepsilon)$, since $x_2' = x_3$. From Eq. (4.4) $x_2 < 2$, hence $-8 - 2\varepsilon + \gamma_0/(1 + \varepsilon) < 2$, so that $\gamma_0 < (10 + 2\varepsilon)(1 + \varepsilon) \leq 24$ at $\varepsilon \geqslant 1$. Inequalities (4.3) are thus proven.

It is now sufficient simply to prove that at τ, greater than τ_0, $x_2 > 1$. In view of the fact that $x_3 > 0$, the inequalities $x_3' < x_2^2 - 4$ and $x_3 < \gamma_0 + (x_2^2 - 4)\tau$ are true. Considering Eq. (4.3), we obtain $x_3 < C_1 + (x_2^2 - 4)\tau$, so that $C_1 + (x_2^2 - 4)\tau > 0$. Choosing $\tau_0 = C_1/3$ and considering that $x_2(\tau) > x_2(\tau_0)$ at $\tau > \tau_0$, we arrive at the required inequality

$$x_2 > 1 \quad \text{at} \quad \tau \geqslant \tau_0 = C_1/3. \tag{4.5}$$

With consideration of inequalities (4.4) the third equation of Eq. (4.2) gives $dx_3/d\tau + x_1x_3 \leqslant 0$, i.e., $x_3(\tau) \leqslant \gamma_0 \exp\left(-\int_0^\tau x_1(s)\,ds\right)$. Integrating the last equation of Eq. (4.2) with consideration of Eq. (4.5), we obtain the inequality $x_1(\tau) > \tau/2$ at $\tau \geqslant \tau_0$, so that

$$x_3(\tau) \leqslant C_3 \exp(-C_4\tau^2) \quad \text{at} \quad \tau \geqslant \tau_0. \tag{4.6}$$

Integrating the second equation of Eq. (4.2) with consideration of Eq. (4.6) and the boundary condition at infinity, we obtain

$$0 \leqslant 2 - x_2 \leqslant C_5 \exp(-C_6\tau^2). \tag{4.7}$$

Restoring the old variables in Eq. (4.1), instead of inequalities (4.7), we obtain the inequality $0 \leqslant 2c - y_2 \leqslant C_5 c \exp(-C_6(t-a)^2/\varepsilon^2)$, which is valid for $t \geqslant a(1 + \tau_0/\sqrt{R})$.

By Eq. (3.2), $y_2 = -f' - f/r$, so that

$$0 \leqslant 2c + f' + f/r \leqslant C_5 c \exp(-C_6(r-a)^2/\varepsilon^2). \tag{4.8}$$

The latter inequality determines the behavior of the function $h(r) = f' + f/r$ as $r \to \infty$ $\nu \to 0$. Integrating Eq. (4.8) with consideration of $f(a) = 0$ gives

$$0 \leqslant f + cr - ca^2/r \leqslant C_7\varepsilon \quad \text{at} \quad r \geqslant a(1 + \varepsilon\tau_0). \tag{4.9}$$

It now remains for us to describe the behavior of the function $g(r)$, which in view of Eqs. (1.2), (1.5) is defined ambiguously. However, the solution of the problem of Eqs. (1.2), (1.5) within the class of functions falling off at infinity more rapidly than const/r is unique, and has the form

$$g(r) = \frac{\omega a^2}{r} \int_r^a \rho \exp\left(\int_a^\rho \frac{f(s)}{\nu} ds\right) d\rho \Big/ \int_a^\infty \rho \exp\left(\int_a^\rho \frac{f(s)}{\nu} ds\right) d\rho,$$

whence, with Eq. (4.9), we obtain $|g| \leqslant C_8 \exp(-C_9 r^2/\nu)$ at $r \geqslant a(1 + \tau_0/\sqrt{R})$.

Thus, as $R \to \infty$ the rotational component of the velocity is localized in a boundary layer with thickness of the order of magnitude of $1/\sqrt{R}$.

5. Asymptotic Expansion in a Small Parameter. We will define an asymptotic representation of the solution of Eqs. (1.1), (1.6), (2.4) by the method of merging inner and outer expansions. Let

$$f(r) = ca\Phi(y), \quad y = r/a, \quad \varepsilon = R^{-1/2}. \tag{5.1}$$

Then Eq. (1.1) may be rewritten in the form

$$\varepsilon^2(\Phi''' + 2\Phi''/y - \Phi'/y^2 + \Phi/y^3) - \Phi\Phi'' + \Phi'^2 + \Phi\Phi'/y + 2\Phi^2/y^2 - 4 = 0. \tag{5.2}$$

Having defined the inner variable $\varepsilon F(\tau) = \Phi(y)$, $y = 1 + \varepsilon\tau$, from Eq. (5.2) we obtain the equation

$$F''' + 2\varepsilon F''/(1+\varepsilon\tau) - \varepsilon^2 F'/(1+\varepsilon\tau)^2 + \varepsilon^3 F/(1+\varepsilon\tau)^3 - FF'' + F'^2$$
$$+ \varepsilon FF'/(1+\varepsilon\tau) + 2\varepsilon^2 F^2/(1+\varepsilon\tau)^2 - 4 = 0.$$

We use an asymptotic representation

$$\Phi(y) = \sum_{n=0}^{\infty} \varepsilon^{2n}\Phi_n(y), \quad F(\tau) = \sum_{n=0}^{\infty} \varepsilon^n F_n(\tau).$$

At $n = 0$ $\Phi_0(y)$, $F_0(\tau)$ are determined as solutions of the problems

$$F_0''' - F_0F_0'' + F_0'^2 - 4 = 0, \quad F_0(0) = F_0'(0) = 0,$$
$$F_0'(\tau) \to \Phi_0'(1) \quad \text{as} \quad \tau \to \infty \tag{5.3}$$

(condition for merger of inner and outer expansions);

$$-\Phi_0\Phi_0'' + \Phi_0'^2 + \Phi_0\Phi_0'/y + 2\Phi_0^2/y^2 - 4 = 0,$$
$$\Phi_0(1) = 0, \quad \Phi_0(y) \to -1 \quad \text{as} \quad y \to \infty. \tag{5.4}$$

We note that the last problem of Eq. (5.4) has a solution $\Phi_0(y) = -y + 1/y$, so that Eq. (5.3) with the condition $F_0'(\tau) \to -2$ as $\tau \to \infty$ has a unique solution [1]. According to Eq. (5.1), we have $f_0(r) = ca\Phi_0(y) = -cr + ca^2/r$. Then inequality (4.9) shows that $|f(r) - f_0(r)| \leqslant C_7\varepsilon$ at $\varepsilon \leqslant 1$, $r \geqslant a(1 + \varepsilon\tau_0)$

In conclusion, we note that although the solution presented here does not describe the real liquid flow, it can be useful in studying liquid flows produced by rotation of a cylinder of finite, but large (in comparison to the radius) length l. We will offer the hypothesis that the solution studied here is the major term of the "inner expansion" of the exact solution near the cylinder as $\nu \to 0$ and large l/a, at least far from the cylinder faces.

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DETERMINATION OF THE BOUNDARY OF A HYDRODYNAMIC CONTACT REGION

M. A. Galakhov and V. P. Kovalev

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The thickness of a lubricating film and the integral hydrodynamic contact force characteristics are determined to a significant degree by the form and dimensions of the contact region [1-5]. The present study will formulate conditions on the boundary of a planar contact region with consideration of surface tension; the problem of boundary determination is formulated within the framework of Reynolds equations.

1. Boundary Conditions for Reynolds Equations. We will consider the flow of a thin liquid layer, separating two surfaces S_1 and S_2 (Fig. 1). We denote by Ω the region within which the liquid occupies the entire interval between the surfaces. Since the layer is thin, in correspondence to Ω we will consider a surface S, lying within Ω at equal distances from S_1 and S_2. We denote by $\gamma \in S$ the boundary of the continuous liquid layer. We will consider the nonstationary problem. Let Ω, S_1, S_2, S, γ depend on time. Each point of $M \in \gamma$ can be described by a moving Cartesian coordinate system $M\xi\eta\zeta$ with unit vectors $\mathbf{n}$, $\boldsymbol{\tau}$, $\mathbf{k}$ such that the vector $\mathbf{k}$ is perpendicular to S, $\boldsymbol{\tau}$ is tangent to γ, and $\mathbf{n}$ is tangent to S and perpendicular to γ, directed outward from Ω. Let $\mathbf{u}_1$ and $\mathbf{u}_2$ be the projections of the velocities of the surfaces S_1 and S_2 on S. We will term the boundary an input (γ_+), if $(\mathbf{n}, \mathbf{u}_1) \leqslant 0$, $(\mathbf{n}, \mathbf{u}_2) \leqslant 0$, $(\mathbf{n}, \mathbf{u}_1)^2 + (\mathbf{n}, \mathbf{u}_2)^2 \neq 0$, an output ($\gamma_-$), if $(\mathbf{n}, \mathbf{u}_1) \geqslant 0$, $(\mathbf{n}, \mathbf{u}_2) \geqslant 0$, $(\mathbf{n}, \mathbf{u}_1)^2 + (\mathbf{n}, \mathbf{u}_2)^2 \neq 0$, or mixed ($\gamma_\pm$), if the conditions for γ_+ and γ_- are not fulfilled. In normal applications boundaries are usually either input or output.

We will assume that the flow in Ω is described by a Reynolds equation, which requires two boundary conditions on the entire free boundary γ. Analysis of the Stokes equation near γ with consideration of surface tension on the boundary between the liquid and surrounding medium shows that if we neglect inertial terms and mass forces and assume the flow to be locally independent of coordinate η, the boundary conditions will have the following structure:

$$p = \frac{2\sigma}{h} p_+ \left[\frac{\sigma}{\mu(\mathbf{u}, \mathbf{n})}, \frac{(\mathbf{u}_2 - \mathbf{u}_1, \mathbf{n})}{(\mathbf{u}, \mathbf{n})}, \frac{h_1}{h}, \frac{h_2}{h}\right]; \tag{1.1}$$

$$(\mathbf{q}_i - \mathbf{q}_0, \mathbf{n}) = 0 \tag{1.2}$$

on γ_+,

$$p = \frac{2\sigma}{h} p_\pm \left[\frac{\sigma}{\mu(\mathbf{u}_1, \mathbf{n})}, -\frac{(\mathbf{u}_2, \mathbf{n})}{(\mathbf{u}_1, \mathbf{n})}, \frac{h_1}{h}\right]; \tag{1.3}$$

$$(\mathbf{q}_i, \mathbf{n}) = h(\mathbf{u}_1, \mathbf{n})\, g\left[\frac{\sigma}{\mu(\mathbf{u}_1, \mathbf{n})}, -\frac{(\mathbf{u}_2, \mathbf{n})}{(\mathbf{u}_1, \mathbf{n})}, \frac{h_1}{h}\right] \tag{1.4}$$

on $\gamma_\pm$ at $(\mathbf{u}_1, \mathbf{n}) < 0$,

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